Int J Theor Phys (2010) 49: 413–420
DOI 10.1007/s10773-009-0218-4

The Implementation of Impulse Approximation in the Wave Function and the Response Function of Many-Fermion System

M. Modarres · Y. Younesizadeh

Received: 3 October 2009 / Accepted: 7 December 2009 / Published online: 15 December 2009

Abstract In this work, we investigate the ambiguities proposed by Benhar et al. about different implementation of the impulse approximation (IA) for calculation of the response function of many-Fermion system. The many-Fermion wave-function of composite system is calculated in the framework of impulse approximation by considering the iteration equation of many-Fermion wave-function through the system Hamiltonian propagator, and it is shown that by imposing the plane wave approximation for the struck particle it is possible to remove these ambiguities (the plane wave impulse approximation (PWIA)). Finally it is concluded that in order to get relevant result, one should be careful to perform the IA on the many-Fermion wave function to calculate the response function of the system, since the system response is obviously very sensitive to this quantity.

Keywords Impulse approximation · Response function · Spectral function · One-body momentum distribution

1 Introduction

A few years ago, Benhar et al. [1] (BFF) showed that within the impulse approximation (IA), the dynamical response function of a $\mathcal{N}$-Fermion system, $\mathcal{S}(\mathbf{q},\omega)$, at large momentum transfer, with ground state energy $\mathcal{E}_0^{\mathcal{N}}$ and Hamiltonian $\mathcal{H}$, can be written in a transparent form in terms of the target *spectral function*, $\mathcal{P}(\mathbf{k},\varepsilon)$ or the widely used alternative definition, the *one-body momentum distribution*, $n(k)$. Furthermore, they concluded that there are considerable differences between the resulting response functions if one applies the above two different definition to the many-Fermion systems.

Recently, similar to BFF proposal, it was also demonstrated by us [2] that the minimal use of impulse approximation leads to the *spectral function* definition of the response

M. Modarres (✉)
Physics Department, University of Tehran, 1439955961, Tehran, Iran
e-mail: mmodares@ut.ac.ir

Y. Younesizadeh
Physics Department, University of Tarbiat-Modarres, Tehran, Iran

function, and it is possible to calculate the approximate condition, $[\mathcal{E}_n^{\mathcal{N}-1} - \mathcal{E}_0^{\mathcal{N}}]_{av} \simeq \epsilon_k$, in which $\mathcal{E}_n^{\mathcal{N}-1}$ is the nth state energy of $\mathcal{N} - 1$ Fermion system and $\epsilon_k = \frac{\hbar^2 k^2}{2m}$, to remove the ambiguities claimed by BFF. It was also shown that the coherent response function is important in low momentum transfer, but it still should be considered in order to satisfy the quantum fluids sum rules. On the other hand, the nucleon-nucleon interaction has dramatic effect on the coherent response function in the low momentum transfer ($q \leq 3\ \text{fm}^{-1}$), while the coherent response function can be ignored for the momentum transfer such as 5 fm^{-1} or larger. Several numerical aspects of one-body momentum distribution and dynamical response function in connection to the nuclear correlation function have been also investigated by us [3–6] and other groups [7–10].

In this work we intend to look into above ambiguities from another point of view, namely the $\mathcal{N}$-Fermion ground state energy and wave function. This can be done through an iteration procedure by imposing the impulse approximation in each step. Obviously, the many-body correlations have crucial effect on the dynamical response function, since the initial momentum of the struck particle is fixed by the *one-body momentum distribution* of the target and there is a final-state interaction (FSI) between the struck particle and the target residue or the spectator. In the impulse approximation (IA) the above correlation and interaction are ignored, i.e. $\mathcal{N} - 1$ spectator particles do not correlate and interact with the struck particle. This is true in the IA, since the delivered momentum by the probe is large enough that the probe sees the target as a collection of individual constituent. But in case of nuclear system, because of strong two-body interaction, even at large momentum transfer, the FSI could still be important [11]. So we will consider these two approximations in our iterated many-body wave-functions to verify above discrepancies.

So the paper will be organized as follow. Section 2 is devoted to the calculation of the ground state energy and wave function of many Fermion system by using the iteration equation in terms of the Hamiltonian propagator, in the IA and the consideration of ambiguities in using this approximation. The discussion and conclusion are presented in Sect. 3.

2 The Calculation of the Ground State Energy and Wave Function in the IA

To impose the first condition of the IA, the Hamiltonian of $\mathcal{N}$-Fermion system is usually divided into three parts [1, 2],

$$\mathcal{H} = \sum_{i=1}^{\mathcal{N}} \mathcal{T}_i + \sum_{j>i=1}^{\mathcal{N}} V_{ij} = \mathcal{H}_0 + \mathcal{T}_1 + \mathcal{H}_{\text{FSI}}, \tag{1}$$

where $\mathcal{H}_0$, $\mathcal{T}_i = \frac{-\nabla_i^2}{2m}$, V_{ij} and m are the $\mathcal{N} - 1$ body spectator,

$$\mathcal{H}_0 = \sum_{i=2}^{\mathcal{N}} \frac{-\nabla_i^2}{2m} + \sum_{j>i=2}^{\mathcal{N}} V_{ij}, \tag{2}$$

the one-body kinetic energy, the two-body interaction and the constituent mass, respectively. In (1),

$$\mathcal{H}_{\text{FSI}} = \sum_{j=2}^{\mathcal{N}} V_{1j} \tag{3}$$

is the final state interactions (FSI) between the hit constituent and the $\mathcal{N}-1$ particle spectator of the system.

In the second assumption of IA, we neglect $\mathcal{H}_{\text{FSI}} \simeq 0$, which means the probe see the struck particle freely. In other words in the IA we calculate all of the quantities of system from the probe point of view. Thus, finally we have:

$$\mathcal{H} \simeq \mathcal{H}_{\text{IA}} = \mathcal{H}_0 + \mathcal{T}_1. \tag{4}$$

For above Hamiltonian we can also write down the corresponding Schrödinger equation,

$$\mathcal{H}_{\text{IA}}|0\rangle = \mathcal{H}_0|0\rangle + \mathcal{T}_1|0\rangle = [\mathcal{E}_0^{\mathcal{N}}]_{\text{IA}}|0\rangle, \tag{5}$$

and similarly for $\mathcal{N}-1$ particle system:

$$\mathcal{H}_0|0\rangle = \mathcal{E}_0^{(\mathcal{N}-1)}|0\rangle. \tag{6}$$

Since $\mathcal{H}_0$ and $\mathcal{T}_1$ commute with each other, so $|0\rangle$ is the eigenstate of $\mathcal{T}_1$ as well and we have,

$$\mathcal{T}_1|0\rangle = \varepsilon_k|0\rangle, \tag{7}$$

with

$$\varepsilon_k = \frac{k^2}{2m}. \tag{8}$$

So for the ground state of $\mathcal{N}$ Fermion system we have,

$$\mathcal{H}_{\text{IA}}|0\rangle = \mathcal{E}_0^{(\mathcal{N}-1)}|0\rangle + \varepsilon_k|0\rangle = (\mathcal{E}_0^{(\mathcal{N}-1)} + \varepsilon_k)|0\rangle, \tag{9}$$

or

$$[\mathcal{E}_0^{\mathcal{N}}]_{\text{IA}} = \mathcal{E}_0^{(\mathcal{N}-1)} + \varepsilon_k. \tag{10}$$

This is the ground state energy of $\mathcal{N}$ Fermion system in the IA, from the point of view of the probe particle.

Now we continue this section by writing the response function that has been defined in terms of the *spectral function* $\mathcal{P}(k,\varepsilon)$,

$$\mathcal{S}(q,\omega) = \int \frac{d\mathbf{k}}{(2\pi)^3} d\varepsilon \mathcal{P}(\mathbf{k},\varepsilon)\delta(\omega - \varepsilon_{|\mathbf{k}+\mathbf{q}|} - \varepsilon), \tag{11}$$

and the *one body momentum distribution*, $n(k)$ [12–16],

$$\mathcal{S}(q,\omega) = \int \frac{d\mathbf{k}}{(2\pi)^3} n(k)\delta(\omega - \varepsilon_{|\mathbf{k}+\mathbf{q}|} + \varepsilon_k), \tag{12}$$

which has created the ambiguities regarding of the response function of many body systems [1, 2]. $\mathcal{P}(k,\varepsilon)$ and $n(k)$ are also related to each other through the following equation:

$$n(k) = \int d\varepsilon \mathcal{P}(\mathbf{k},\varepsilon), \tag{13}$$

with the following definitions:

$$\mathcal{P}(\mathbf{k},\varepsilon) = \sum_n \left| \int d\mathbf{R} e^{i\mathbf{k}.\mathbf{r}_1} \Psi_0^*(\mathbf{R})\Phi_n(\widetilde{\mathbf{R}}) \right|^2 \delta(\varepsilon + \mathcal{E}_0^{\mathcal{N}} - \mathcal{E}_n^{(\mathcal{N}-1)}), \tag{14}$$

and

$$n(k)=\int d\mathbf{r_1}d\acute{\mathbf{r}}_{\mathbf{1}}e^{i\mathbf{k}.(\mathbf{r_1}-\acute{\mathbf{r}}_{\mathbf{1}})}\int d\widetilde{\mathbf{R}}\Psi_0^*(\mathbf{r_1},\widetilde{\mathbf{R}})\Psi_0(\acute{\mathbf{r}}_{\mathbf{1}},\widetilde{\mathbf{R}}), \tag{15}$$

where we have $\mathbf{R}\equiv(\mathbf{r_1},\mathbf{r_2},\dots,\mathbf{r_N})$, $\widetilde{\mathbf{R}}\equiv(\mathbf{r_2},\dots,\mathbf{r_N})$, $\Psi_0(\mathbf{R})$ is the ground state wave function of $\mathcal{N}$ particle system and $\mathcal{E}_n^{\mathcal{N}-1}$ satisfy the $\mathcal{N}-1$ Schrödinger equation with eigenfunctions $\Phi_n(\widetilde{\mathbf{R}})$. Note that there is no approximation regarding equations of (14) and (15) in terms of the system many-body wave function.

Let us now, to calculate the *spectral function* in the IA. So first we start by calculating the iteration equation for $\Psi_0(\mathbf{R})=\langle\mathbf{R}|0\rangle$ and the propagator $\langle\mathbf{R}|\exp(i\mathcal{H}t)|\acute{\mathbf{R}}\rangle$ in this approximation. Since we have

$$e^{i\mathcal{H}t}|0\rangle=e^{i\mathcal{E}_0^{\mathcal{N}}t}|0\rangle, \tag{16}$$

so

$$|0\rangle=\frac{e^{i\mathcal{H}t}}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}|0\rangle. \tag{17}$$

We substitute (17) into $\Psi_0(R)$ which leads to:

$$\Psi_0(\mathbf{R})=\langle\mathbf{R}|0\rangle=\langle\mathbf{R}|\frac{e^{i\mathcal{H}t}}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}|0\rangle=\frac{1}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}\int d\acute{\mathbf{R}}\langle\mathbf{R}|e^{i\mathcal{H}t}|\acute{\mathbf{R}}\rangle\langle\acute{\mathbf{R}}|0\rangle. \tag{18}$$

Thus we have:

$$\Psi_0(\mathbf{R})=\frac{1}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}\int d\acute{\mathbf{R}}\langle\mathbf{R}|e^{i\mathcal{H}t}|\acute{\mathbf{R}}\rangle\Psi_0(\acute{\mathbf{R}}), \tag{19}$$

which is the iteration equation in terms of the propagator $\langle\mathbf{R}|\exp(i\mathcal{H}t)|\acute{\mathbf{R}}\rangle$. To test this equation, the propagator itself can be expanded as,

$$\langle\mathbf{R}|e^{i\mathcal{H}t}|\acute{\mathbf{R}}\rangle=\sum_n\langle\mathbf{R}|e^{i\mathcal{H}t}|n\rangle\langle n|\acute{\mathbf{R}}\rangle=\sum_n e^{i\mathcal{E}_n t}\Psi_n(\mathbf{R})\Psi_n^*(\acute{\mathbf{R}}), \tag{20}$$

where in (20) we have used the Schrödinger equation, $\mathcal{H}|n\rangle=\mathcal{E}_n^{(\mathcal{N})}|n\rangle$. By inserting (20) into (19) we get,

$$\frac{1}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}\int d\acute{\mathbf{R}}\sum_n e^{i\mathcal{E}_n t}\Psi_n(\mathbf{R})\Psi_n^*(\acute{\mathbf{R}})\Psi_0(\acute{\mathbf{R}})=\frac{1}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}\sum_n e^{i\mathcal{E}_n t}\Psi_n(\mathbf{R})\int d\acute{\mathbf{R}}\Psi_n^*(\acute{\mathbf{R}})\Psi_0(\acute{\mathbf{R}}). \tag{21}$$

Then by using the orthonormality relation for the wave functions i.e. $\int d\acute{\mathbf{R}}\Psi_n^*(\acute{\mathbf{R}})\Psi_0(\acute{\mathbf{R}})=\delta_{n,0}$, we will have:

$$\frac{1}{e^{i\mathcal{E}_0^{\mathcal{N}}t}}\sum_n e^{i\mathcal{E}_n^{\mathcal{N}}t}\Psi_n(\mathbf{R})\delta_{n,0}=\Psi_0(\mathbf{R}). \tag{22}$$

So (19) is in general exact. As we pointed out before (19) is the equation for iteration of wave function to calculate the ground state wave function and we indent to show that different approximation on this equation leads to different definition of response function in terms of the *spectral function* and the *one-body momentum distribution* (see (14) and (15)). We proceed by writing the propagator in right hand side of the above equation (19) in the IA [1]:

$$\langle\mathbf{R}|e^{i\mathcal{H}t}|\acute{\mathbf{R}}\rangle\simeq\langle\mathbf{R}|e^{i\mathcal{H}_{\mathrm{IA}}t}|\acute{\mathbf{R}}\rangle=\langle\widetilde{\mathbf{R}}|e^{i\mathcal{H}_0 t}|\widetilde{\acute{\mathbf{R}}}\rangle\langle\mathbf{r_1}|e^{i\mathcal{T}_1 t}|\acute{\mathbf{r}}_{\mathbf{1}}\rangle. \tag{23}$$

Now substitution of the above propagators into (19) it will give us:

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{1}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \int d\acute{\mathbf{R}} \langle \widetilde{\mathbf{R}} | e^{i\mathcal{H}_0 t} | \acute{\widetilde{\mathbf{R}}} \rangle \langle \mathbf{r_1} | e^{i\mathcal{T}_1 t} | \acute{\mathbf{r}}_\mathbf{1} \rangle \Psi_0^{\mathrm{IA}}(\acute{\mathbf{R}}). \tag{24}$$

But similar to (20), the two propagators in (24) can be expanded as,

$$\langle \widetilde{\mathbf{R}} | e^{i\mathcal{H}_0 t} | \acute{\widetilde{\mathbf{R}}} \rangle = \sum_n e^{i\mathcal{E}_n^{(\mathcal{N}-1)} t} \Phi_n(\widetilde{\mathbf{R}}) \Phi_n^*(\acute{\widetilde{\mathbf{R}}}), \tag{25}$$

and

$$\langle \mathbf{r_1} | e^{i\mathcal{T}_1 t} | \acute{\mathbf{r}}_\mathbf{1} \rangle = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\varepsilon_p t} e^{i\mathbf{p}.(\mathbf{r_1} - \acute{\mathbf{r}}_\mathbf{1})}, \tag{26}$$

where $\Phi_n(\widetilde{\mathbf{R}}) = \langle \widetilde{\mathbf{R}} | n \rangle$, $\mathcal{H}_0 | n \rangle = \mathcal{E}_n^{(\mathcal{N}-1)} | n \rangle$ and $\varepsilon_p = \frac{p^2}{2m}$.

The substitution of (25) and (26) in (24) leads to:

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{1}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \int d\acute{\mathbf{R}} \sum_n e^{i\mathcal{E}_n^{(\mathcal{N}-1)} t} \Phi_n(\widetilde{\mathbf{R}}) \Phi_n^*(\acute{\widetilde{\mathbf{R}}}) \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\varepsilon_p t} e^{i\mathbf{p}.(\mathbf{r_1} - \acute{\mathbf{r}}_\mathbf{1})} \Psi_0^{\mathrm{IA}}(\acute{\mathbf{R}}). \tag{27}$$

We also have different orthonormality conditions namely:

$$\int d\widetilde{\mathbf{R}} \Phi_n^*(\widetilde{\mathbf{R}}) \Phi_m(\widetilde{\mathbf{R}}) = \delta_{n,m}, \tag{28}$$

and

$$\sum_n \Phi_n^*(\widetilde{\mathbf{R}}) \Phi_n(\acute{\widetilde{\mathbf{R}}}) = \delta(\widetilde{\mathbf{R}} - \acute{\widetilde{\mathbf{R}}}). \tag{29}$$

Here we can *not* easily use the above orthonormality relations to simplify (27), as the one we did in (21). But one can easily find out that, this equation can be simplified only if we separate the struck particle wave function from the spectator system wave function and accept following relation, i.e.

$$\Psi_0^{\mathrm{IA}}(\acute{\mathbf{R}}) = \phi_0(\acute{\mathbf{r}}_\mathbf{1}) \Phi_0(\acute{\widetilde{\mathbf{R}}}). \tag{30}$$

This assumption is totally true in the IA by means of $H_{\mathrm{FSI}} = 0$. But we must also make another approximation and substitute $\phi_0(\acute{\mathbf{r}})$ with:

$$\phi_0(\acute{\mathbf{r}}) = \frac{1}{v^{\frac{1}{2}}} e^{i\mathbf{k}.\acute{\mathbf{r}}}, \tag{31}$$

which is called the *plane wave impulse approximation* (PWIA) [17]. With these two assumptions we can continue our simplification of (27) as:

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{1}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \int d\acute{\mathbf{R}} \sum_n e^{i\mathcal{E}_n^{\mathcal{N}-1} t} \Phi_n(\widetilde{\mathbf{R}}) \Phi_n^*(\acute{\widetilde{\mathbf{R}}}) \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\varepsilon_p t} e^{i\mathbf{p}.(\mathbf{r_1} - \acute{\mathbf{r}}_\mathbf{1})} \phi_0(\acute{\mathbf{r}}_\mathbf{1}) \Phi_0(\acute{\widetilde{\mathbf{R}}}), \tag{32}$$

or

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{1}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \int d\acute{\mathbf{r}}_\mathbf{1} \sum_n e^{i\mathcal{E}_n^{\mathcal{N}-1} t} \Phi_n(\widetilde{\mathbf{R}}) \int d\acute{\widetilde{\mathbf{R}}} \Phi_n^*(\acute{\widetilde{\mathbf{R}}}) \Phi_0(\acute{\widetilde{\mathbf{R}}})$$
$$\times \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\varepsilon_p t} e^{i\mathbf{p}.(\mathbf{r_1} - \acute{\mathbf{r}}_\mathbf{1})} \phi_0(\acute{\mathbf{r}}_\mathbf{1}). \tag{33}$$

Now by using (28):

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{1}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \int d\acute{\mathbf{r}}_\mathbf{1} e^{iE_0^{\mathcal{N}-1}t} \Phi_0(\widetilde{\mathbf{R}}) \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\varepsilon_p t} e^{i\mathbf{p}.(\mathbf{r_1}-\acute{\mathbf{r}}_\mathbf{1})} \phi_0(\acute{\mathbf{r}}_\mathbf{1}). \tag{34}$$

Then by substituting $\phi_0(\acute{\mathbf{r}}) = \frac{1}{v^{\frac{1}{2}}} e^{i\mathbf{k}.\acute{\mathbf{r}}}$ where $\mathbf{k}$ is the struck particle momentum of the ground state, we find,

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{e^{i\mathcal{E}_0^{\mathcal{N}-1}t}}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \Phi_0(\widetilde{\mathbf{R}}) \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\varepsilon_p t} e^{i\mathbf{p}.\mathbf{r_1}} \frac{1}{v^{\frac{1}{2}}} \int d\acute{\mathbf{r}}_\mathbf{1} e^{-i(\mathbf{p}-\mathbf{k}).\acute{\mathbf{r}}_\mathbf{1}}. \tag{35}$$

But we have $\int d\acute{\mathbf{r}}_\mathbf{1} e^{-i(\mathbf{p}-\mathbf{k}).\acute{\mathbf{r}}_\mathbf{1}} = (2\pi)^3\delta(\mathbf{p}-\mathbf{k})$, so,

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{e^{i\mathcal{E}_0^{\mathcal{N}-1}t}}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \Phi_0(\widetilde{\mathbf{R}}) e^{i\varepsilon_k t} \frac{1}{v^{\frac{1}{2}}} e^{i\mathbf{k}.\mathbf{r_1}}, \tag{36}$$

or

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{e^{i[\mathcal{E}_0^{\mathcal{N}-1}+\varepsilon_k]t}}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \Phi_0(\widetilde{\mathbf{R}}) \phi_0(\mathbf{r_1}). \tag{37}$$

We can also insert from (10) into above equation and get the correct result,

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}}{e^{i[\mathcal{E}_0^{\mathcal{N}}]_{\mathrm{IA}}t}} \Phi_0(\widetilde{\mathbf{R}}) \phi_0(\mathbf{r_1}) = \Psi_0^{\mathrm{IA}}(\mathbf{R}). \tag{38}$$

Thus finally with the substitution of,

$$\Psi_0^{\mathrm{IA}}(\mathbf{R}) = \frac{1}{v^{\frac{1}{2}}} e^{i\mathbf{k}.\acute{\mathbf{r}}} \Phi_0(\widetilde{\acute{\mathbf{R}}}), \tag{39}$$

in (27), the iteration equation (19) works correctly for the IA case which is valid if one ignores FSI interaction [1, 2].

Now if we substitute (30) into (14) and apply (10), we will have:

$$\mathcal{P}_{\mathrm{IA}}(\mathbf{k},\varepsilon) = \sum_n \left| \int d\mathbf{R} e^{i\mathbf{k}.\mathbf{r_1}} \phi_0^*(\mathbf{r_1}) \Phi_0^*(\widetilde{\mathbf{R}}) \Phi_n(\widetilde{\mathbf{R}}) \right|^2 \delta(\varepsilon + \mathcal{E}_0^{\mathcal{N}-1} + \varepsilon_k - \mathcal{E}_n^{\mathcal{N}-1}), \tag{40}$$

or

$$\mathcal{P}_{\mathrm{IA}}(\mathbf{k},\varepsilon) = \sum_n \left| \int d\mathbf{r_1} e^{i\mathbf{k}.\mathbf{r_1}} \phi_0^*(\mathbf{r_1}) \int d\widetilde{\mathbf{R}} \Phi_0^*(\widetilde{\mathbf{R}}) \Phi_n(\widetilde{\mathbf{R}}) \right|^2 \delta(\varepsilon + \mathcal{E}_0^{\mathcal{N}-1} + \varepsilon_k - \mathcal{E}_n^{\mathcal{N}-1}). \tag{41}$$

But by applying the orthonormality of $\Phi_n(\widetilde{\mathbf{R}})$ we get,

$$\mathcal{P}_{(\mathrm{IA})}(\mathbf{k},\varepsilon) = \left| \int d\mathbf{r_1} e^{i\mathbf{k}.\mathbf{r_1}} \phi_0^*(\mathbf{r_1}) \right|^2 \delta(\varepsilon + \varepsilon_k), \tag{42}$$

or

$$\mathcal{P}_{(\mathrm{IA})}(\mathbf{k},\varepsilon) = \int d\mathbf{r_1} d\acute{\mathbf{r}}_\mathbf{1} e^{i\mathbf{k}.(\mathbf{r_1}-\acute{\mathbf{r}}_\mathbf{1})} \phi_0^*(\mathbf{r_1}) \phi_0(\acute{\mathbf{r}}_\mathbf{1}) \int d\widetilde{\mathbf{R}} \Phi_0^*(\widetilde{\mathbf{R}}) \Phi_0(\widetilde{\mathbf{R}}) | \delta(\varepsilon + \varepsilon_k), \tag{43}$$

where we have again used (28). Finally we get,

$$\mathcal{P}_{(\mathrm{IA})}(\mathbf{k},\varepsilon)=\int d\mathbf{r_1}d\acute{\mathbf{r}}_\mathbf{1}e^{i\mathbf{k}.(\mathbf{r_1}-\acute{\mathbf{r}}_\mathbf{1})}\int d\widetilde{\mathbf{R}}\Psi_0^{*\mathrm{IA}}(\mathbf{r_1},\widetilde{\mathbf{R}})\Psi_0{}^{\mathrm{IA}}(\acute{\mathbf{r}}_\mathbf{1},\widetilde{\mathbf{R}})\delta(\varepsilon+\varepsilon_k), \tag{44}$$

which can easily be written in terms of the *one-body momentum distribution*, (15),

$$\mathcal{P}_{(\mathrm{IA})}(\mathbf{k},\varepsilon)=n(k)_{\mathrm{IA}}\delta(\varepsilon+\varepsilon_k). \tag{45}$$

Note that in (44) the ground state wave function should be written in the PWIA.

If as an example, we consider non-interacting Fermi gas model of the *one body momentum distribution*, which is simply one for $k\le k_F$ and zero for $k>k_F$ (k_F is Fermi momentum) and substitute it in (45) we will have [7–10]:

$$\mathcal{P}_{(IA)}(\mathbf{k},\varepsilon)=\delta(\varepsilon+\varepsilon_k). \tag{46}$$

Indeed (46) is the *spectral function* in the PWIA [17].

On the other hand, we can substitute (45) in (11) and simply get (12) by performing integration over ε:

$$\mathcal{S}(\mathbf{q},\omega)=\int\frac{d\mathbf{k}}{2\pi^3}d\varepsilon\; n(k)\delta(\varepsilon+\varepsilon_k)\delta(\omega-\varepsilon_{|\mathbf{k}+\mathbf{q}|}-\varepsilon), \tag{47}$$

that is,

$$\mathcal{S}(\mathbf{q},\omega)=\int\frac{d\mathbf{k}}{2\pi^3}n(k)\delta(\omega+\varepsilon_k-\varepsilon_{|\mathbf{k}+\mathbf{q}|}). \tag{48}$$

So, the above calculations confirm this matter that there are not any ambiguities for the response function in the PWIA. The cause of ambiguities is that we do not calculate all of quantities in the same order in the IA.

3 Summary and Conclusion

In this work, we considered the impulse approximation (IA) to calculate the *spectral function* and *one body momentum distribution* of many-Fermion system. We calculated the ground state wave function and ground state energy in the IA by using the iteration equation through the related Hamiltonian propagator. It was shown that the response function in terms of *spectral function* becomes equal to the corresponding response function defined in terms of *one body the momentum distribution* if we perform the similar approximation in the many-Fermion wave function. Since the IA considers the ground state energy and wave function of the system from point of view of the probe particle, one should calculate both the ground state wave function and the ground state energy in the same order in the IA. One should note that these quantities change in comparison with the state of system without approximation and this behavior is natural. The *spectral function* which has been calculated in the IA displays this fact that the residual system has roughly lost only the kinetic energy of the struck particle ($-\frac{k^2}{2m}$) from point of view of probe particle [11]. Finally our calculation still agrees with BFF [1] and MY [2] that minimal use of the IA leads to the definition of response function in terms of the *spectral function*, while according to the present work with further approximation (the PWIA) one can obtain the same results in terms of the *one body momentum distribution*.

Acknowledgements We would like to thank the Research Council of University of Tehran and Institute for Research and Planning in Higher Education for the grants provided for us.

References

1. Benhar, O., Fabrocini, A., Fantoni, S.: Phys. Rev. Lett. **87**, 052501 (2001)
2. Modarres, M., Younesizadeh, Y.: Nucl. Phys. A **789**, 82 (2007)
3. Modarres, M.: J. Phys. G, Nucl. Phys. **13**, 755 (1987)
4. Modarres, M.: Europhys. Lett. **3**, 1083 (1987)
5. Modarres, M., Moeini, H., Moshfegh, H.R.: Mod. Phys. Lett. A **20**, 1903 (2005)
6. Modarres, M., Rajabi, A.: Eur. Phys. J. B (2009, in press)
7. Benhar, O., Fabrocini, A., Fantoni, S., Sick, I.: Nucl. Phys. A **579**, 493 (1994)
8. Fantoni, S., Pandharipande, V.R.: Nucl. Phys. A **427**, 473 (1984)
9. Benhar, O., Fabrocini, A., Fantoni, S.: Nucl. Phys. A **505**, 267 (1989)
10. Carraro, C., Koonin, S.E.: Phys. Rev. B **41**, 6741 (1989)
11. Benhar, O., Day, D., Sick, I.: Rev. Mod. Phys. **80**, 189 (2008)
12. Czyz, W., Gottfried, K.: Nucl. Phys. **21**, 676 (1961)
13. Czyz, W., Gottfried, K.: Ann. Phys. (NY) **21**, 47 (1963)
14. West, G.B.: Phys. Rep. **10**, 2 (1975)
15. Antonov, A.N., Hodgson, P.E., Petkov, I.Zh.: Nucleon Momentum and Density Distributions in Nuclei. Clarendon Press, Oxford (1988)
16. Antonov, A.N., Hodgson, P.E., Petkov, I.Zh.: Nuclear Correlation in Nuclei. Springer, Heidelberg (1993)
17. Marchand, C., et al.: Phys. Rev. Lett. **60**, 1703 (1988)